

1. A 0.35 kg ball is connected to a string and is swung in a vertical circle of radius 2.3 metres at a constant speed of 5.6 m/s.

a. What is the period?

$$v = \frac{d}{t} \rightarrow t = \frac{d}{v} = \frac{2\pi(2.3\text{m})}{5.6\text{m/s}}$$

$$= 2.581\text{sec}$$

$$\approx 2.6\text{sec}$$

b. What is the frequency?

$$f = \frac{1}{T} = \frac{1}{2.581\text{sec}} = 0.3875\text{Hz} \approx 0.39\text{Hz}$$

c. What is the tension in the string at the top of the circle?



$$F_c = F_g + \text{tension}$$

$$\frac{mv^2}{r} = mg + T$$

~~d. What is the tension in the string at the bottom of the circle?~~

$$\frac{(0.35\text{kg})(\overset{5.6}{\cancel{2.581}})^2}{2.3} = 0.35\text{kg} \times \frac{9.8\text{m}}{\text{s}^2} + T$$

$$4.772\text{N} = 3.43\text{N} + T$$

$$4.772 - 3.43 = T$$

$$= 1.34\text{N} = T \approx \boxed{1.3\text{N}}$$

~~2. What is the minimum speed a 5.2 kg mass can be swung in a vertical circle of radius 0.25m?~~

What is the tension in the string at bottom of circle?



$$F_c = T - F_g$$

$$4.772\text{N} = T - 3.43\text{N}$$

$$4.772\text{N} + 3.43\text{N} = T$$

$$\boxed{8.2\text{N} = T}$$

1. A 0.35 kg ball is connected to a string and is swung in a vertical circle of radius 2.3 metres at a constant speed of 5.6 m/s.

- a. What is the period?
- b. What is the frequency?
- c. What is the tension in the string at the top of the circle?
- d. What is the tension in the string at the bottom of the circle?

2. What is the minimum speed a 5.2 kg mass can be swung in a vertical circle of radius 0.25m?

At top $T=0$ and $F_c = F_g$

$$\frac{mv^2}{r} = mg$$

$$v^2 = rg$$

$$v = \sqrt{rg} = \sqrt{0.25\text{m} \times 9.8\frac{\text{m}}{\text{s}^2}}$$

$= 1.6\text{m/s}$

3. What is the maximum speed a 5.2 kg mass can be swung in a vertical circle of radius 0.25m if the string holding the mass can withstand a maximum tension of 85 N?

Tension will be greatest at the bottom

$$F_c = T - F_g = 85 - 5.2 \times 9.8 = 34.04 \text{ N}$$

$$F_c = \frac{mv^2}{r} \rightarrow \sqrt{\frac{F_c r}{m}} = v$$

$$\sqrt{\frac{34.04 \times 0.25}{5.2}} = 1.3 \text{ m/s}$$

~~2.9 m/s~~

4. Two equal masses are swung at the same constant speed around circles of different radii. If one mass has two times the centripetal force acting on it compared to the other, how does its circle's radius compare to the other?

$$F_{c1} = 2 F_{c2}$$

$$m \frac{v^2}{r_1} = 2 \left(m \frac{v^2}{r_2} \right)$$

$$\frac{1}{r_1} = \frac{2}{r_2}$$

$$r_2 = 2 r_1 \rightarrow r_1 = \frac{1}{2} r_2$$

the one with twice the F_c has a radius half as big

5. Two equal masses are swung around circles of the same radius. One is swung twice as fast as the other. How do the centripetal forces acting on the masses compare?

$$v_1 = 2v_2$$

~~one~~

F_c of larger

$$\frac{mv_1^2}{r}$$

$$(2v_2)^2$$

$$4v_2^2$$

F_c of smaller

$$\frac{mv_2^2}{r}$$

$$v_2^2$$

$$v_2^2$$

One with twice the velocity has 4 times the F_c

6. A 0.65 kg ball is swung in a vertical circle, it makes a revolution every 0.56 seconds.
 a. What is the period?

$$0.56 \text{ sec}$$

- b. What is the frequency?

$$f = \frac{1}{T} = \frac{1}{0.56 \text{ sec}} = 1.7857 \text{ Hz} \approx 1.8 \text{ Hz}$$

- c. Is the centripetal force acting on the ball the same throughout the circle?

Yes, if it is in uniform circular motion

- d. What is the centripetal force acting on the ball?

$$F_c = \frac{m 4 \pi^2 r}{T^2} = \frac{(0.65 \text{ kg})(4)(\pi)^2 \times (0.40 \text{ m})}{(0.56)^2}$$

$$= 32.73 \text{ N} \approx \textcircled{33 \text{ N}}$$

- e. What is the tension in the string at the top of the circle?

$$F_c = T + F_g \rightarrow T = F_c - F_g$$

$$= 32.73 \text{ N} - 0.65 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2}$$

$$= 26.36 \text{ N} \approx \textcircled{26 \text{ N}}$$

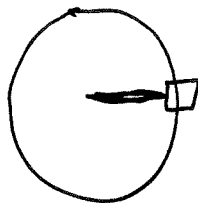
- f. What is the tension in the string at the bottom of the circle?

$$F_c = T - F_g \rightarrow T = F_c + F_g = 39.1 \text{ N}$$

$$\approx \textcircled{39 \text{ N}}$$

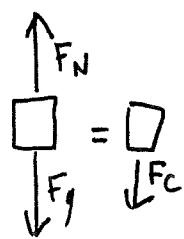
- g. What is the tension in the string when the ball is halfway between the top and bottom of the circle?

$$T = F_c = 32.73 \text{ N} \approx \textcircled{33 \text{ N}}$$



7. A 64 kg person is in a Ferris wheel with radius of 8.0m which makes one complete revolution every 25 seconds.

a. What is their apparent weight at the top of the ride?

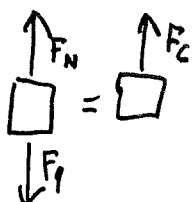


$$F_c = \frac{m 4\pi^2 r}{T^2} = \frac{64 \times 4 \times \pi^2 \times 8}{25^2} = 32.34 \text{ N} \leftarrow \text{this is the not force acting on them downwards}$$

$$F_g = mg = 64 \times 9.8 = 627.2 \text{ N}$$

$$F_c = F_g - F_N \rightarrow F_N = F_g - F_c = 627.2 \text{ N} - 32.34 \text{ N} = 594.86 \text{ N} \approx 590 \text{ N}$$

b. What is their apparent weight at the bottom of the ride?



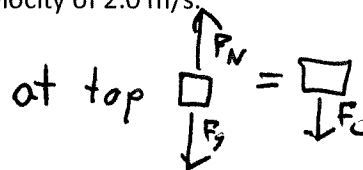
$$F_c = F_N - F_g \rightarrow F_N = F_c + F_g = 32.34 \text{ N} + 627.2 \text{ N} = 659.54 \text{ N} \approx 660 \text{ N}$$

8. A 45 kg person is in a Ferris wheel with radius of 18.0m which moves at a velocity of 2.0 m/s.

a. What is their apparent weight at the top of the ride?

$$F_c = \frac{mv^2}{r} = \frac{45 \text{ kg} \times (2 \text{ m/s})^2}{18.0 \text{ m}} = 10 \text{ N}$$

$$F_N = F_g - F_c = mg - 10 = 441 \text{ N} - 10 \text{ N} = 431 \text{ N} \approx 430 \text{ N}$$

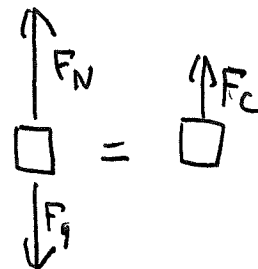


F_c is downwards

b. What is their apparent weight at the bottom of the ride?

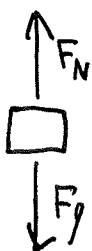
$$F_N = F_N + F_g = 441 + 10 = 451 \text{ N}$$

at bottom F_c is pointing upwards



9. A 950 kg car is driving over the top of a hill which can be viewed as part of a circle of radius 820m. What is the maximum speed the car can drive over the hill without losing contact between the car and the ground?

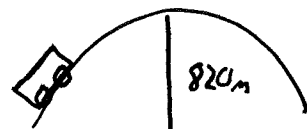
It will lose contact when there is no normal force between the car and the road



At max speed at top $F_N = 0$

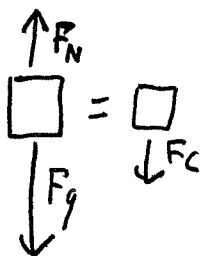
$$\text{so } F_c = F_g \rightarrow \frac{mv^2}{r} = mg$$

$$v = \sqrt{rg} = \sqrt{820\text{m} \times 9.8\frac{\text{m}}{\text{s}^2}} = 89.6\text{m/s} \approx 9.0 \times 10^1 \text{m/s}$$



10. A 1200 kg car is driving over the top of a hill which can be viewed as part of a circle of radius 250m. They travel at a speed of 25 m/s.

- a. What is the normal force acting on the car?



$$F_c = F_g - F_N$$

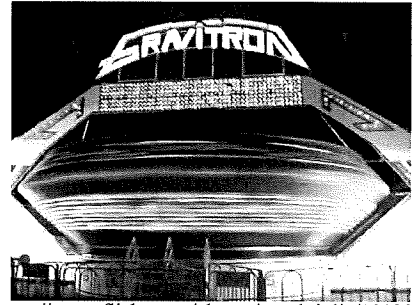
$$F_N = F_g - F_c$$

$$= 1200\text{kg} \times 9.8\frac{\text{m}}{\text{s}^2} - \frac{1200 \times 25^2}{250} = 8760\text{N} \approx 8800\text{N}$$

- b. What is the force of friction acting on the car at the top of the hill if coefficient of friction between the car and the road is 0.35?

$$F_{\text{fric}} = \mu F_N = 0.35 \times 8760\text{N} \\ = 3066\text{N} \\ \approx 3100\text{N}$$

11. The Graviton is a classic amusement park ride. When you get in, you stand against the wall, 5.0 m from the centre of the ride and then the whole ride starts spinning quickly. At the max speed each revolution takes 1.6 seconds. A 45 kg person gets on the ride.



<https://www.flickr.com/photos/unrulyjulie/10198149>

- a. What is the centripetal force acting on the person when it is at max speed?

$$F_c = \frac{m 4\pi^2 r}{T^2} = \frac{45 \text{ kg} \times 4 \times \pi^2 \times 5.0 \text{ m}}{1.6^2}$$

$$= 3469.78 \text{ N}$$

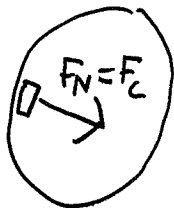
$$\approx 3500 \text{ N}$$

- b. If while spinning the wall suddenly fell away, what would happen to the person?

They would fly outwards

- c. What is the normal force the wall is applying to the person?

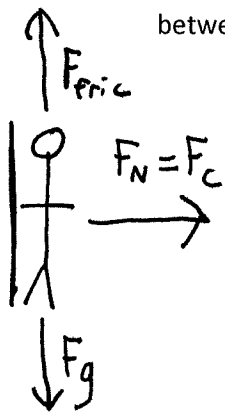
Normal force is the force keeping the person moving in circle



$$F_N = F_c = 3469.78 \text{ N}$$

$$\approx 3500 \text{ N}$$

12. In some versions of the Gravitron the floor drops away during the ride and people are held onto the wall by the force of friction between them and the wall. What is the minimum speed the ride could spin so that a 65 kg person could be held by the wall if the coefficient of friction between them and the wall is 0.22? *and distance to centre is 5.0m*



If they are held $F_{\text{fric}} = F_g$

$$F_{\text{fric}} = 65 \text{ kg} \times 9.8 \text{ m/s}^2 = 637 \text{ N}$$

$$F_{\text{fric}} = \mu F_N \rightarrow F_N = \frac{F_{\text{fric}}}{\mu} = \frac{637 \text{ N}}{0.22} = 2895.45 \text{ N}$$

$$F_N = F_c \rightarrow \frac{mv^2}{r} = F_N \rightarrow v = \sqrt{\frac{F_N \times r}{m}}$$

$$= \sqrt{\frac{2895.45 \times 5.0 \text{ m}}{65 \text{ kg}}} = 14.92 \text{ m/s}$$

$$= \boxed{15 \text{ m/s}}$$

13. A 2.3 gram dime is placed on a merry-go-round, which is spun with period of 7.0 seconds, the dime moves out from where it was initially placed but then stays on the spinner a certain distance from the centre. If the coefficient of friction between the dime and the merry-go-round is 0.19, how far from the centre does it end up?



https://commons.wikimedia.org/wiki/File:Tift_Park_merry-go-round.JPG

At final position $F_c = F_{\text{fric}}$

$$\frac{m 4\pi^2 r}{T^2} = \mu mg \rightarrow 4\pi^2 r = \mu g T^2$$

$$r = \frac{\mu g T^2}{4\pi^2}$$

$$= \frac{0.19 \times 9.8 \times 7^2}{4\pi^2}$$

$$= \boxed{2.3 \text{ m}}$$

14. A student stands 1.2 m from the centre of a merry go round, what is the minimum period it can be spun with so that the student doesn't slide if they have a coefficient of friction of 0.34 with the merry go round? *Note that mass will cancel out.

$$F_c = F_{\text{fric}}$$

$$\cancel{m} \frac{4\pi^2 r}{T^2} = \mu \cancel{m} g$$

$$\sqrt{\frac{4\pi^2 r}{\mu g}} = T \rightarrow \sqrt{\frac{4\pi^2(1.2)}{0.34 \times 9.8}} = 3.77 \text{ sec}$$

$$\approx 3.8 \text{ sec}$$

15. Flying a fighter plane involves a lot of force being applied to the pilot. Often, we speak of the g-force acting on the pilot, however g-force isn't really a force, it is an acceleration, 1 g-force is 9.8 m/s². What is the g-force acting on a pilot if they are experiencing an acceleration of 17 m/s²?

~~17 m/s²~~

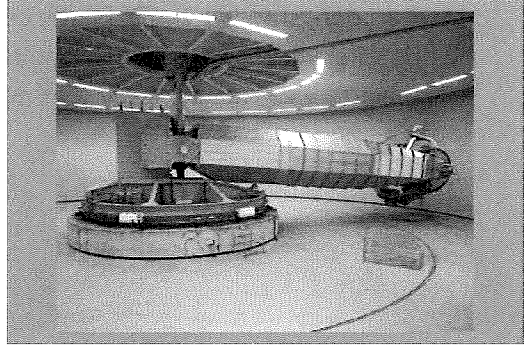
$$17 \text{ m/s}^2 \times \frac{1g}{9.8 \text{ m/s}^2} = 1.7g \text{ - Force}$$

16. A pilot is spun in a training centrifuge of radius 4.50 metres, experiencing 3.41 g-force. What is the period of this rotation?

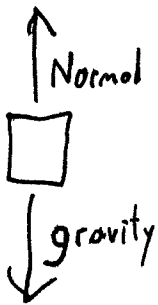
$$3.41g \times 9.8 = 33.418 \text{ m/s}^2$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2(4.50\text{m})}{33.418\text{m/s}^2}} = 2.3 \text{ sec}$$



17. What is the ^{maximum} acceleration (in g's) provided by the normal force acting on a pilot who is flying a vertical circle of radius 750 m at a velocity of 260 m/s?



$$a_c = \frac{v^2}{r} = \frac{260^2}{750} = 90.13 \text{ m/s}^2$$

At the bottom centripetal acceleration provided by normal force is partly cancelled out by acceleration due to gravity, so normal is greatest at that point

acceleration due to normal at

$$\text{bottom} = 90.13 \text{ m/s}^2 + 9.8 \text{ m/s}^2$$

$$= 99.93 \text{ m/s}^2$$

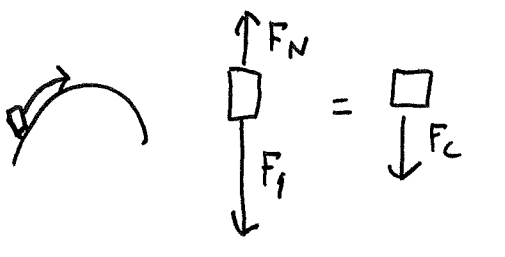
$$\div 9.8$$

$$= 10.2 \text{ g-force}$$

$$\approx 1.0 \times 10^1 \text{ g-force}$$

18. A 1200 kg car goes over the top of a small hill which can be modelled as part of a circle of radius 250 m, while at the same time making a turn around a circle of radius 250 m. They are travelling at 25 m/s. What is the minimum coefficient of friction between the car and the ground so that they can successfully make the turn?

① Determine normal force at top of hill



$$F_c = \frac{mv^2}{r} = 5000 \text{ N}$$

$$F_c = \overset{\text{win}}{F_g} - \overset{\text{Lose}}{F_N} = 11760 \text{ N} - 5000 \text{ N}$$

$$= 6760 \text{ N}$$

② Determine F_{fric} during turn

$$F_{\text{fric}} = F_c = \frac{mv^2}{r} = \frac{1200 \times 25^2}{250} = 3000 \text{ N}$$

③ Determine μ

$$F_{\text{fric}} = \mu F_N \rightarrow \mu = \frac{F_{\text{fric}}}{F_N} = \frac{3000}{6760} = 0.44$$

b) What would the minimum coefficient be if the turn was on flat ground

$$\mu = \frac{F_{\text{fric}}}{F_N} = \frac{F_{\text{fric}}}{F_g} = \frac{3000 \text{ N}}{11760 \text{ N}} = 0.26$$

19. One problem of space travel is the lack of gravity and its effects on the bones of people. One way to generate artificial gravity is to have the whole space ship rotate to simulate Earth's gravity. If a space ship is a 550 m radius disc and the astronauts will walk around the inside of the disc with their heads towards the centre, what speed must it spin at to simulate Earth's gravity of 9.8 m/s^2 ?

$$a_c = \frac{v^2}{r} \rightarrow \sqrt{ra_c} = v$$
$$\sqrt{550 \text{ m} \times 9.8 \frac{\text{m}}{\text{s}^2}} = v$$
$$73 \text{ m/s} = v$$

- 20) Tension is greatest at lowest point
since it must overcome gravity to
equal F_c

Breaks at bottom